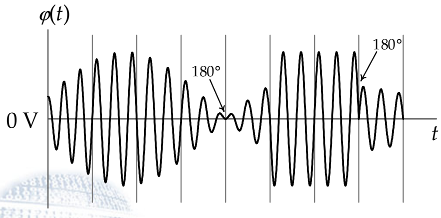
Lecture 15: Frequency & Phase Modulation (FM & PM)

Prof. Mohammed Hawa
Electrical Engineering Department
The University of Jordan

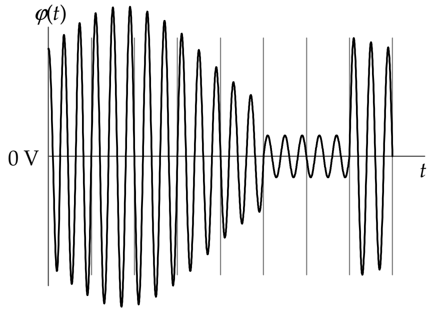
EE421: Communications I: Lecture 15. For more information read Chapter 5 in your textbook or visit http://wikipedia.org/.

Remember: Amplitude Modulation

$\varphi(t) = m(t)\cos(\omega_c t)$



$\varphi(t) = [m(t) + A]\cos(\omega_c t)$



Angle Modulation (FM & PM)

$$c(t) = A\cos(\omega_c t + \theta_0)$$

$$\varphi_{FM\ or\ PM}(t) = A\cos\theta(t)$$

$$\omega_i(t) \triangleq \frac{d\theta(t)}{dt}$$

$$\theta_i(t) \triangleq \theta(t) - \omega_c t$$

$\theta(t)$: **generalized angle** of modulated signal.
$\omega_i(t)$: **instantaneous frequency** of modulated signal.
$\theta_i(t)$: **instantaneous phase** of modulated signal.

 Electrical Engineering Department, The University of Jordan 3

Frequency Modulation (FM)

- The *instantaneous frequency* of the modulated signal changes in proportion to the message.

$$\varphi_{FM}(t) = A\cos\left(\omega_c t + k_f\int_{-\infty}^{t} m(t)dt\right)$$

$$\theta_{FM}(t) = \omega_c t + k_f\int_{-\infty}^{t} m(t)dt$$

$$\omega_{i_{FM}}(t) = \omega_c + k_f m(t)$$

$$\theta_{i_{FM}}(t) = k_f\int_{-\infty}^{t} m(t)dt$$

 Electrical Engineering Department, The University of Jordan 4

Phase Modulation (PM)

- The *instantaneous phase* of the modulated signal changes in proportion to the message.

$$\varphi_{PM}(t) = A\cos\left(\omega_c t + k_p m(t)\right)$$

$$\theta_{PM}(t) = \omega_c t + k_p m(t)$$

$$\theta_{i_{PM}}(t) = k_p m(t)$$

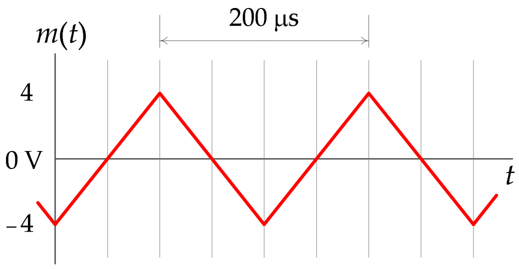
$$\omega_{i_{PM}}(t) = \omega_c + k_p \frac{dm(t)}{dt} = \omega_c + k_p m'(t)$$

FM and PM Equivalence

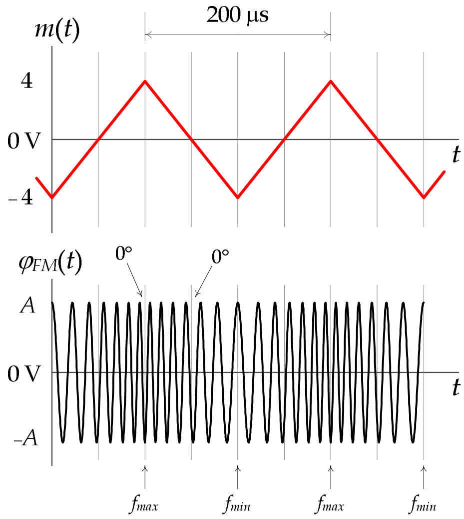
- FM
  - Constant amplitude A
  - Constant carrier frequency ω_c
  - Variable *instantaneous* frequency $\omega_i \propto m(t)$
  - Variable *instantaneous* phase $\theta_i \propto \int m(t)dt$
- PM
  - Constant amplitude A
  - Constant carrier frequency ω_c
  - Variable *instantaneous* frequency $\omega_i \propto m'(t)$
  - Variable *instantaneous* phase $\theta_i \propto m(t)$

Example 1

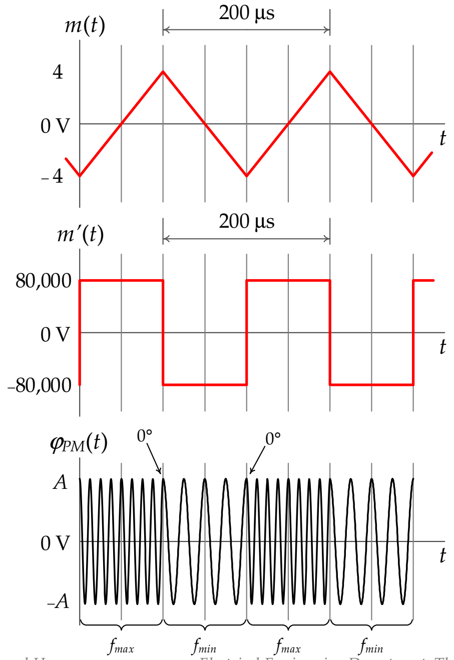
- For the following message signal $m(t)$ and a 100 MHz carrier:
  a) Sketch the **FM** modulated signal. Use $k_f = 2\pi\times10^5$ rad/s/V.
  b) Sketch the **PM** modulated signal. Use $k_p = 5\pi$ rad/V.
  c) Find Δf for both modulated signals.



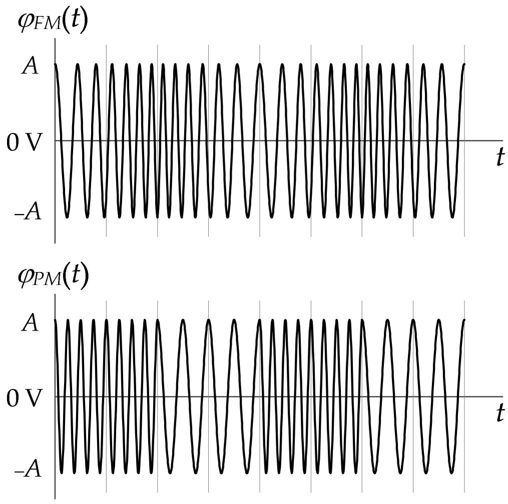
Solution: FM



PM


m(t)
200 μs
4
0 V
–4
t
m′(t)
200 μs
80,000
0 V
–80,000
t
φPM(t)
0°
0°
A
0 V
–A
t
fmax
fmin
fmax
fmin

FM vs. PM


φFM(t)
A
0 V
–A
t
φPM(t)
A
0 V
–A
t

Peak Frequency Deviation

- For **FM**:

$$\Delta f \triangleq \frac{f_{max} - f_{min}}{2} = \frac{k_f}{2\pi} \times \frac{m(t)_{max} - m(t)_{min}}{2}$$

$$\Delta f = \frac{k_f}{4\pi} \times m(t)_{pk-pk} \quad [Hz]$$

- For **PM**:

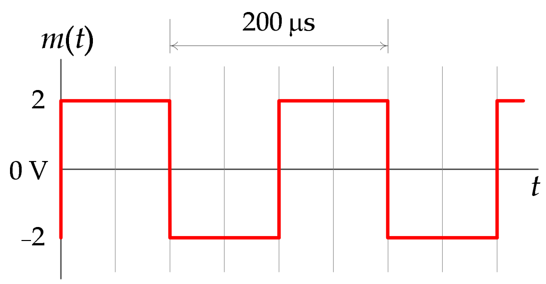
$$\Delta f \triangleq \frac{f_{max} - f_{min}}{2} = \frac{k_p}{2\pi} \times \frac{m'(t)_{max} - m'(t)_{min}}{2}$$

$$\Delta f = \frac{k_p}{4\pi} \times m'(t)_{pk-pk} \quad [Hz]$$

 Electrical Engineering Department, The University of Jordan 11

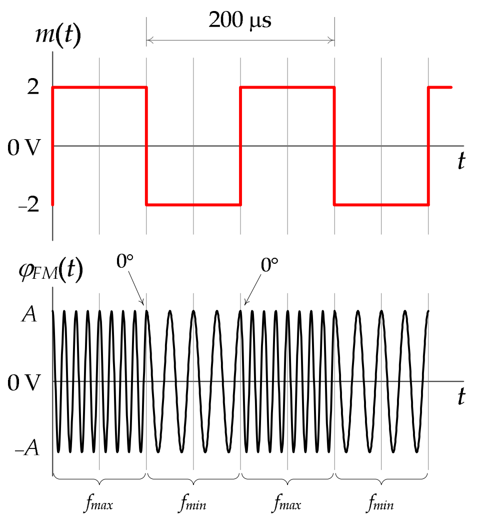
Example 2

- For the following message signal $m(t)$ and a 100 MHz carrier:
  a) Sketch the **FM** modulated signal. Use $k_f = 2\pi \times 10^4$ rad/s/V.
  b) Sketch the **PM** modulated signal. Use $k_p = \pi/4$ rad/V.
  c) Find Δf for both modulated signals.



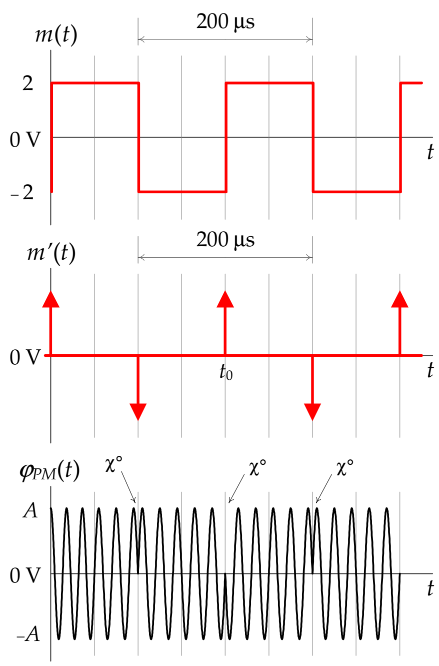
 Electrical Engineering Department, The University of Jordan 12

Solution: FM or FSK



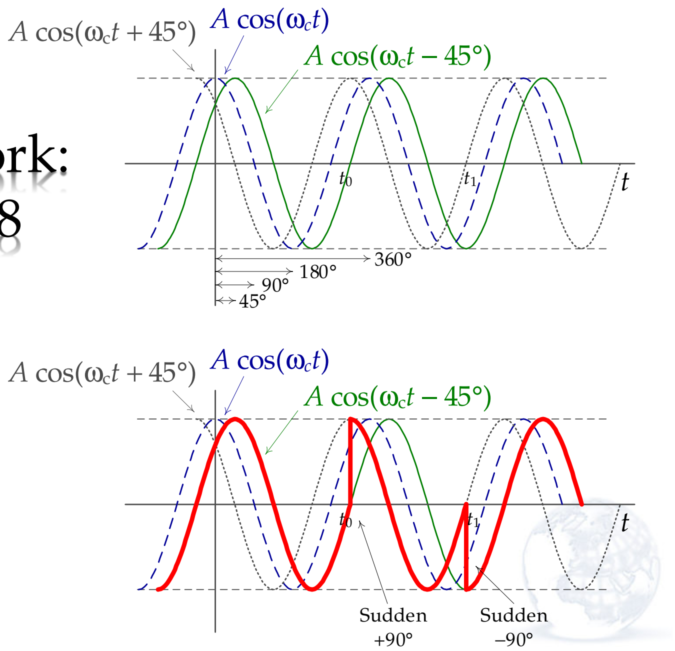
 Electrical Engineering Department, The University of Jordan 13

PM or BPSK



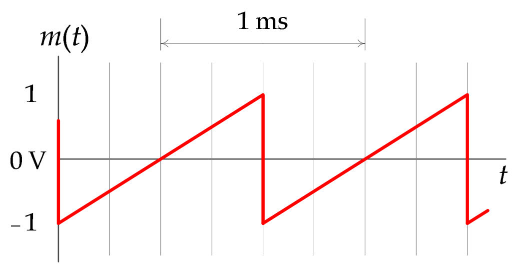
 Electrical Engineering Department, The University of Jordan 14

Homework: $k_p = \pi/8$

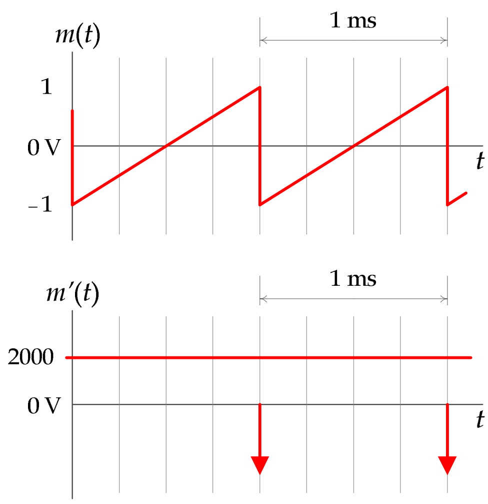


Homework: P.5.1-2

- For the following message signal $m(t)$ and a 200 MHz carrier:
  a) Sketch the **FM** modulated signal. Use $k_f = 2000\pi$ rad/s/V.
  b) Sketch the **PM** modulated signal. Use $k_p = \pi/2$ rad/V.
  c) Try other k_f and k_p values. What is the effect?
  d) Find Δf for both modulated signals.



Hint: For PM



Rules of Thumb

- Smooth change in instantaneous frequency *always* means smooth change in instantaneous phase.
- Sudden change in instantaneous frequency (i.e., unit step change) *does not* mean a sudden change in phase, i.e., it means 0° sudden phase shift.
- *Impulse* change in instantaneous frequency (i.e., infinity frequency) *might* cause a sudden change in phase. To determine the sudden phase shift (or lack thereof) see $k_p m(t)$ for PM or $k_f \int m(t)dt$ for FM.

FM and PM Average Power

$$\phi_{FM}(t) = A\cos\left(\omega_c t + k_f \int_{-\infty}^{t} m(t)dt\right)$$

$$\phi_{PM}(t) = A\cos\left(\omega_c t + k_p m(t)\right)$$

$$\overline{\varphi_{FM}^2(t)} = \frac{A^2}{2}$$

$$\overline{\varphi_{PM}^2(t)} = \frac{A^2}{2}$$

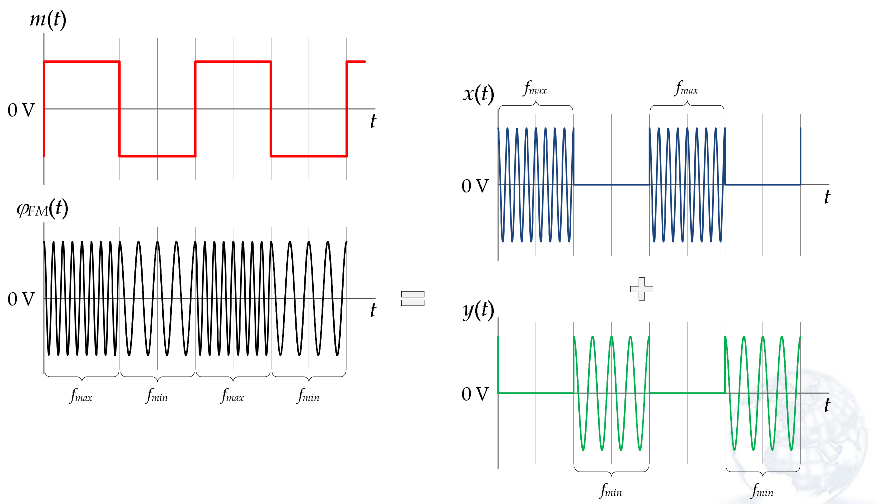
 Electrical Engineering Department, The University of Jordan 19

FM and PM Bandwidth

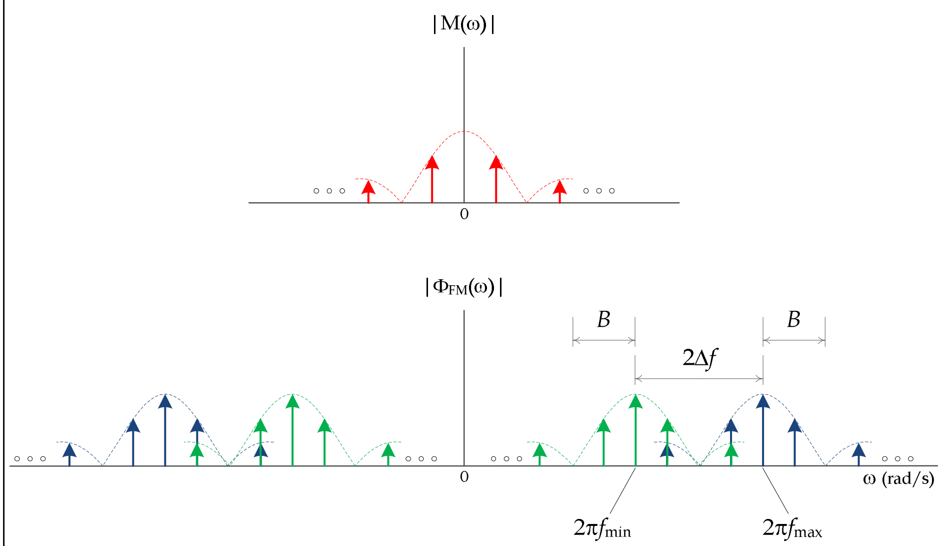
- Mathematically speaking:
  - $B_{\text{FM}} = \infty$
  - $B_{\text{PM}} = \infty$
- Practically speaking, use **Carson's Rule**:
  - $B_{\text{FM}} \approx 2\Delta f + 2B = 2B(\beta+1)$
  - $B_{\text{PM}} \approx 2\Delta f + 2B = 2B(\beta+1)$
- FM Modulation Index:
  - $\beta = \Delta f / B$
  - *Narrow-Band FM* (NBFM) has $\beta \ll 1$ or $\Delta f \ll B$
  - *Wide-Band FM* (WBFM) has $\beta \gg 1$ or $\Delta f \gg B$
  - FM radio uses WBFM with $\beta = 5$

 Electrical Engineering Department, The University of Jordan 20

FM Bandwidth: *Semi-proof*



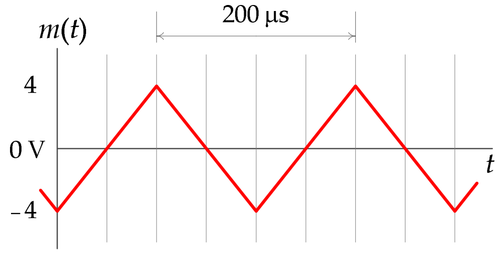
 Electrical Engineering Department, The University of Jordan



 Electrical Engineering Department, The University of Jordan

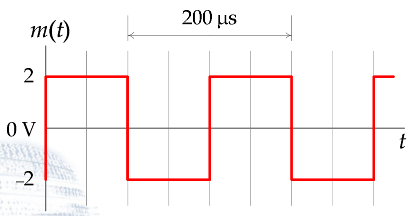
Bandwidth: Example 1

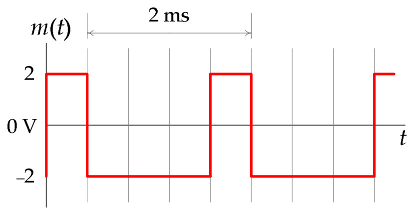
- Estimate the bandwidth B_{FM} and B_{PM} for the modulating signal $m(t)$ shown below. Assume $k_f = \pi\times10^4$ rad/s/V and $k_p = \pi/4$ rad/V.
- *Answers*: B_{FM} = 60 kHz; B_{PM} = 40 kHz.



Bandwidth: Example 2

- Estimate the bandwidth B_{FM} and B_{PM} for the modulating signal $m(t)$ shown below. Assume $k_f = \pi\times10^5$ rad/s/V and $k_p = 5\pi$ rad/V.
- *Answers*: 220 kHz; 20 kHz; 204 kHz; 4 kHz;





FM Signal-to-Noise Ratio

$$SNR_{out} = \left(\frac{3\beta^2}{k_m^2}\right)\frac{S_{in}}{N_0 B}$$

$$S_{in} = k^2\overline{\varphi_{FM}^2(t)} = k^2\left(\frac{A^2}{2}\right)$$

$$k_m^2 = \frac{m_p^2}{\overline{m^2(t)}}$$

Modulation Technique	Modulated Signal Bandwidth	SNR_{out}	Noise Figure NF, dB	Typical Applications
DSB-SC	$2B$	$\frac{S_{in}}{N_0 B}$	-3	Analog instrumentation; multiplexing as part of FM stereo
SSB-SC	B	$\frac{S_{in}}{N_0 B}$	0	Point-to-point voice
VSB-SC	$B\sim 2B$	$\frac{S_{in}}{N_0 B}$	$-3\sim 0$	Facsimile (Fax machines)
QAM	$2B$ for two signals	$\frac{S_{in,effective}}{N_0 B}$	0	Transmit color information in TV broadcasting; digital data
AM	$2B$	$\eta\frac{S_{in}}{N_0 B}$	$-10\log(2\eta)$	Broadcast AM radio; point-to-point voice
SSB+C	B	$\eta\frac{S_{in}}{N_0 B}$	$-10\log(\eta)$	Multiplexing in old telephony systems; point-to-point voice
VSB+C	$B\sim 2B$	$\eta\frac{S_{in}}{N_0 B}$	$-10\log(2\eta)\sim -10\log(\eta)$	Analog Television broadcasting
FM	$2\Delta f + 2B$	$\left(\frac{3\beta^2}{k_m^2}\right)\frac{S_{in}}{N_0 B}$	$10\log\left(\frac{k_m^2}{6(\beta+1)\beta^2}\right)$	Broadcast FM radio; analog microwave links
PM	$2\Delta f + 2B$	$\left(\frac{(\Delta\theta)^2}{k_m^2}\right)\frac{S_{in}}{N_0 B}$	$10\log\left(\frac{k_m^2 B}{2(\Delta\theta)^2(\Delta f + B)}\right)$	Telemetry; digital data

FM (and PM) vs. AM

- FM (and PM) Advantages:
  - FM is capable of exchanging SNR for bandwidth.
  - The constant amplitude of FM makes it less susceptible to nonlinearities.
  - Due to the constant amplitude, FM is less vulnerable than AM to adjacent-channel interference.
  - The constant amplitude of FM gives it a kind of immunity against rapid fading (even with the larger bandwidth).

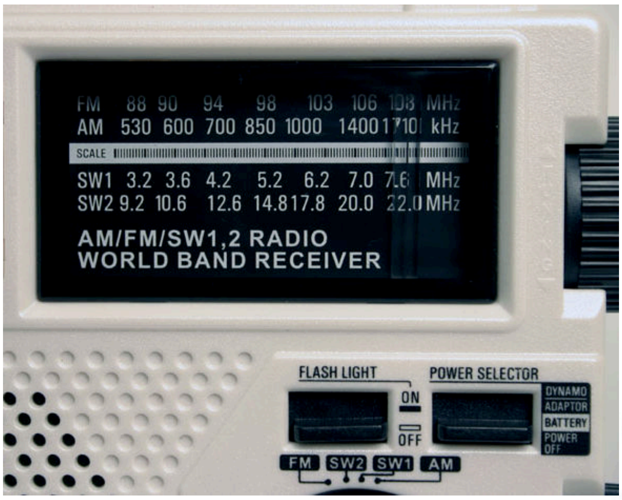
FM (and PM) vs. AM

- FM (and PM) Disadvantages:
  - WBFM (which provides better quality) requires large transmission bandwidth.
  - FM modulators and demodulators are relatively more expensive that AM hardware.
  - PM demodulation requires synchronous detection (relatively expensive).

Applications: FM Radio

- FM + FDM
  - The baseband message is 15 kHz (voice + music).
  - With β = 5, the bandwidth of each FM station is 200 kHz (both U.S. and Europe).
  - The broadcast range is 88 – 108 MHz.
- FM radio sounds better than AM radio:
  - m(t) has a larger bandwidth.
  - WBFM: exchanging SNR for bandwidth.

HW: Look at Your Radio Dial



FM Hardware

- FM Modulator:
  - VCO (Voltage-Controlled Oscillator).
- FM Demodulator:
  - FM Discriminator (also called Slope Detector or Ratio Detector).
  - Quadrature Detector.
  - Phase-Locked Loop.